



# Electro-magneto-thermo-elastic response of infinite functionally graded cylinders without energy dissipation



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## ARTICLE INFO

### Article history:

Received 9 January 2015

Received in revised form

6 July 2015

Accepted 13 July 2015

Available online 14 July 2015

### Keywords:

Finite element method

Infinite cylinder

Electro-magneto-thermo-elastic response

## ABSTRACT

The electro-magneto-thermo-elastic analysis problem of an infinite functionally graded (FG) hollow cylinder is studied in the context of Green–Naghdi's (G–N) generalized thermoelasticity theory (without energy dissipation). Material properties are assumed to be graded in the radial direction according to a novel power-law distribution in terms of the volume fractions of the metal and ceramic constituents. The inner surface of the FG cylinder is pure metal whereas the outer surface is pure ceramic. The equations of motion and the heat-conduction equation are used to derive the governing second-order differential equations. A finite element scheme is presented for the numerical purpose. The system of differential equations is solved numerically and some plots for displacement, radial and electromagnetic stresses, and temperature are presented. The radial displacement, mechanical stresses and temperature as well as the electromagnetic stress are all investigated along the radial direction of the infinite cylinder.

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## 1. Introduction

The classical theory of thermoelasticity has an assumption of infinite speed, which is contrary to physical observation. Various generalized theories of thermoelasticity were developed to replace the classical one of heat conduction in solids. The most important generalized theories of thermoelasticity are Lord and Shulman's (L–S) theory [1], Green and Lindsay's (G–L) theory [2], and Green and Naghdi's (G–N) theory [3–5]. The first two theories (L–S and G–L) introduced one or two relaxation time in the thermoelastic process to eliminate the paradox of infinite speed for the propagation of thermal signals. They also involved a hyperbolic-type heat equation. They are structurally different and one cannot be obtained as a particular case of the other. The third generalized thermoelasticity theory of Green and Naghdi presented the governing thermoelasticity equations in three models. They obtained coupled equations in displacement and temperature fields based on finite wave speed.

The theory of electro-magneto-thermoelasticity is concerned with the interacting effects of the applied electromagnetic field on the elastic and thermoelastic deformations of a solid body. This

theory has aroused much interest in many industrial applications, particularly in nuclear devices, where there exists a primary magnetic field. Various investigations are to be carried out by considering the interactions among electric, magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic and electric studies. The development of the interactions of an electromagnetic field, the thermal field, and the elastic field is available in many studies. Recently, Zenkour and his colleagues [6–10] have presented the analysis of functionally graded piezoelectric cylinders and plates in a hygrothermal environment.

The theory of thermoelasticity without energy dissipation of Green and Naghdi [4] includes the “thermal displacement gradient” among its independent constitutive variables, and differs from the previous theories in that it does not accommodate dissipation of thermal energy. The propagation of thermoelastic waves in different structures is studied on the basis of Green and Naghdi's (G–N) generalized thermoelastic theory (without energy dissipation). Several investigations relating to thermoelasticity without energy dissipation theory have been presented by Chandrasekharaiah [11,12], Sharma and Chouhan [13], Roychoudhuri and Bandyopadhyay [14], Roychoudhuri and Dutta [15]. The coupled thermo-elasticity based on the G–N theory without energy dissipation is developed for infinite and finite functionally graded (FG) thick hollow cylinder using hybrid Galerkin finite element

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